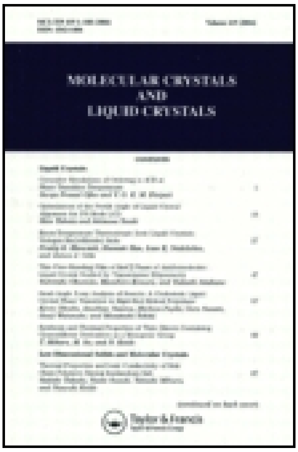

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Discrete Diffractions and Nematicons in Chiral Nematic Liquid Crystals

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Discrete Diffractions and Nematicons in Chiral Nematic Liquid Crystals

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In the following paper we present numerical results of light propagation in a cell filled with low birefringent chiral nematic liquid crystals with a small pitch. Due to the periodicity of a structure typical diffraction is substituted with the discrete diffraction similar to the coupling in waveguide array. For particular anisotropy and appropriate pitch diffraction can be controlled by power. Discussion about the shape and the direction of the launched beam is also provided.

Keywords Chiral nematic liquid crystals; cholesterics; discrete diffraction; light propagation; spatial solitons; nematicons

Introduction

Discrete diffraction was observed and analyzed in many systems for instance in fibers and waveguide arrays and is still actively investigated [1–2–3]. The phenomenon is related to the light coupling for which the first theoretical basis were proposed by Snyder [4] and then confirmed experimentally in linear [5] and nonlinear [6] systems. Recently discrete diffraction was also observed in nematic liquid crystals [7–8–9–10]. These strongly nonlinear materials make it possible to steer discrete diffraction by power. When increasing power diffraction can be confined and form a discrete soliton, in nematic liquid crystals called a nematicon. Solitons were recently also observed in high birefringent chiral nematic liquid crystals (ChNLCs, i.e. cholesterics) [11]. Due to our studies by decreasing the pitch and anisotropy of a chiral nematic it is also possible to obtain discrete diffraction in ChNLCs and control it by power. Some simple theoretical analyses have been already done [12]. In this paper we present more accurate numerical results showing conditions necessary to observe discrete diffraction and discrete solitons in ChNLCs. The vectorial beam propagation method is used for light propagation and calculation of molecules orientation (described by two angles) in a single elastic constant approximation.

The analyzed configuration is presented in Fig. 1. The light beam of a Gaussian shape is launched into the ChNLC cell perpendicularly to the x-axis (a helix axis) and propagates along the z-axis. For the light linearly polarized in y-direction, ChNLC texture is equivalent to the medium with periodically modulated refractive index, where the modulation amplitude is equal to the birefringence of nematics. This can be treated as an array of plane (planar) waveguides and the size of these waveguides is defined by the half of the pitch

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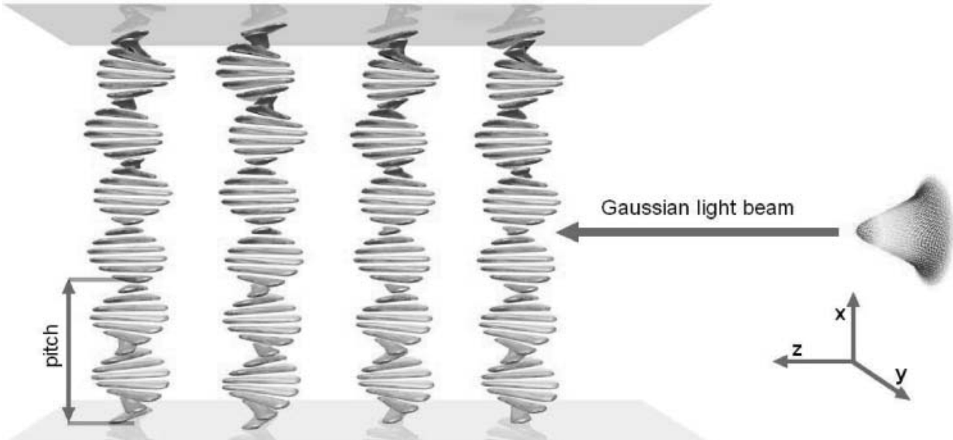

Figure 1. Schematic presentation of a chiral nematic cell.

of ChNLCs. Consequently, the light is guided in thin layers, where the refractive index is the largest, i.e. close to the planes, where molecules are parallel to the electric field of electromagnetic wave (parallel to the y-axis).

For large anisotropy the y-polarized light is confined along the x direction and diffracts in y direction. For higher power the y-polarized light can form a non diffractive beam, a soliton. The x-polarized light is always diffracted. For low anisotropy of the liquid crystals the y-polarized light will be rather diffracted along the x-axis than confined. Due to the periodicity of cholesterics it will be a discrete diffraction similar to the coupling in waveguide array. For particular birefringence such phenomenon can be controlled by power. The higher the input power the narrower the diffraction and thus longer the coupling length is. For particular values a soliton can be formed.

Numerical Methods

To describe light propagation in ChNLCs it is important to model optical field propagation as well as reorientation of the liquid crystals molecules caused by that field. Due to the anisotropy of ChNLCs it is important to use vectorial methods. In simulations Full-Vector Beam Propagation Method (FV-BPM)[13,14] is used without paraxial approximation. We obtain all components of the electric and magnetic fields intensity from the following equations:

$$\begin{aligned}
&\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = i\omega_0\varepsilon_0(\varepsilon_{11}E_x + \varepsilon_{12}E_y + \varepsilon_{13}E_z)\\
&\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = i\omega_0\varepsilon_0(\varepsilon_{21}E_x + \varepsilon_{22}E_y + \varepsilon_{23}E_z)\\
&\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega_0\varepsilon_0(\varepsilon_{31}E_x + \varepsilon_{32}E_y + \varepsilon_{33}E_z)\\
&H_x = -\frac{1}{i\mu_0\omega_0}\cdot\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\\
&H_y = -\frac{1}{i\mu_0\omega_0}\cdot\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\\
&H_z = -\frac{1}{i\mu_0\omega_0}\cdot\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)
\end{aligned} \tag{1}$$

Where H, E—magnetic and electric fields, ε_{ij}—dielectric tensor components.

We start with electric and magnetic field distribution corresponding to a Gaussian beam of the following form

$$
\begin{aligned}
E_x &= E_0 \exp\left(-\frac{x^2+y^2}{4\sigma_0^2}\right)\sin\alpha \\
H_y &= \frac{\sqrt{\varepsilon_{11}}}{Z_0}E_x \\
E_y &= E_0 \exp\left(-\frac{x^2+y^2}{4\sigma_0^2}\right)\cos\alpha \\
H_x &= \frac{\sqrt{\varepsilon_{22}}}{Z_0}E_y
\end{aligned}
$$

where E_0—amplitude, $\sigma_0 = \frac{FWHM}{2\sqrt{2\ln 2}}$—beam waist, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$—impedance of vacuum, α—polarization angle.

We also assume Dirichlet boundary conditions i.e. the electric and magnetic field vanishes at the edges of the system. By using finite differences and solving the equations (1) using Runge-Kutta 4th order scheme we obtain optical field distribution along the whole cell. In nematics the nonlinearity is mainly caused by the molecules reorientation and therefore it is crucial to describe director orientation as well. The following Frank-Oseen equation on free energy density is used [15–16–17]:

$$
\begin{aligned}
f = {} & \frac{1}{2}K_{11}(\nabla\vec{n})^2 + \frac{1}{2}K_{22}(\vec{n}\cdot(\nabla\times\vec{n}) - G)^2 + \frac{1}{2}K_{33}(\vec{n}\times(\nabla\times\vec{n}))^2 + \\
& -\frac{1}{2}\Delta\varepsilon\varepsilon_0(\vec{n}\cdot\vec{E})^2
\end{aligned} \tag{2}
$$

where K_{ii}—are elastic (Frank) constants corresponding to splay, twist and bend deformations, $G = 2\pi/p$ (p-pitch) is a parameter reflecting chirality, $\Delta\varepsilon = \varepsilon_{||} - \varepsilon_{\perp}$—electric anisotropy, $\varepsilon_{||}, \varepsilon_{\perp}$—correspond to the extraordinary and ordinary electric permittivity. The director $\vec{n}$ is defined as: $\vec{n} = \left[\cos\theta;\ \sin\theta\sin\varphi;\ \sin\theta\cos\varphi\right]$ which leads to the dielectric tensor of the following form:

$$
\varepsilon = \begin{bmatrix}
\varepsilon_{\perp} + \Delta\varepsilon\cos^2\theta & \Delta\varepsilon\sin\theta\cos\theta\sin\varphi & \Delta\varepsilon\sin\theta\cos\theta\cos\varphi \\
\Delta\varepsilon\sin\theta\cos\theta\sin\varphi & \varepsilon_{\perp} + \Delta\varepsilon\sin^2\varphi\sin^2\theta & \Delta\varepsilon\sin\varphi\cos\varphi\sin^2\theta \\
\Delta\varepsilon\sin\theta\cos\theta\cos\varphi & \Delta\varepsilon\sin\varphi\cos\varphi\sin^2\theta & \varepsilon_{\perp} + \Delta\varepsilon\sin^2\theta\cos^2\varphi
\end{bmatrix}
$$

The tensor reflects influence of molecules orientation on permittivity and is used in equation (1).

We also assume that all Frank elastic constants are equal $K_{11} = K_{22} = K_{33} = K$.

The solution is obtained by minimizing equation (2) with Euler-Lagrange equations and solving it using Successive Over Relaxation Method [18]

Results

Numerical simulations were performed on a 2D grid of 200 × 200 points propagated along the z-axis. Integration step was set to $\Delta z = 10$ nm and transverse steps were as follows $\Delta x = \Delta y = 250$ nm. Simulations were carried out on a distance of up to 1000 μm. Material parameters such as refractive indices (ordinary refractive index $n_o = 1.456$) and Frank constants ($K = 7.3$ pN) are corresponding to the low birefringent liquid crystals synthesized by prof. Dąbrowski laboratory at Military University of Technology in Warsaw. The wavelength of the input beam was $\lambda = 793$ nm. The input beam was y-polarized

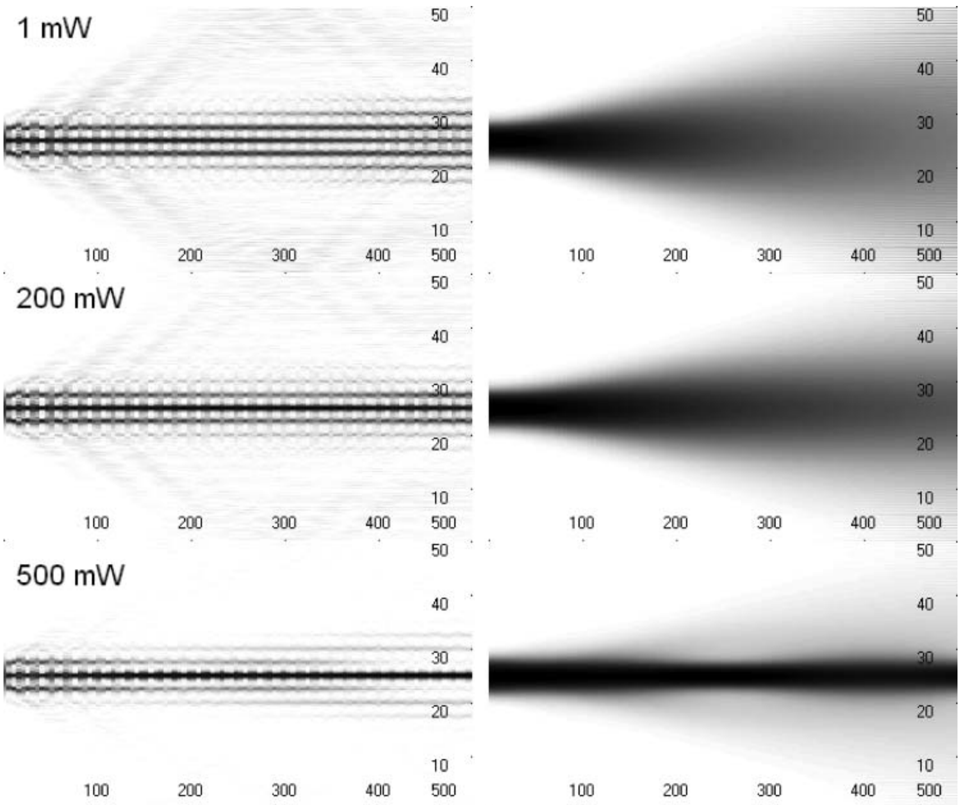

Figure 2. Light propagation for a different input beam power in *xz* plane (left) and *yz* plane (right), anisotropy $\Delta n = 0.06$. Soliton formation is observed for P = 500 mW.

(perpendicular to the helix axis). The cell height was $H = 50\,\mu$m and a pitch of the cholesterics was $p = 5\,\mu$m which gave a total rotation of molecules through an angle of 20π.

Phenomenon of forming a soliton for the beam of $FWHM = 4\,\mu$m and anisotropy $\Delta n = 0.06$ is shown in Fig. 2. By increasing power, optical reorientation of ChNLCs increases the effective refractive index and discrete diffraction in *xz* plane as well as diffraction in *yz* plane can be confined. Discrete diffraction in *xz* plane depends on the coupling length between layers of the cholesterics. In *yz* plane light is propagated like in a free space so its propagation depends on the Rayleigh length. To form two dimensional soliton easily initial diffraction in *x* and *y* directions should be the same, i.e. both, the coupling and Rayleigh lengths should be the same. In other cases soliton formation might be difficult or even impossible, because when the light intensity will be enough to diminish diffraction in one direction then for diminishing in the other direction the intensity will be too low (which causes diffraction, decreasing intensity and consequently diffraction in both directions) or too large (which causes self-focusing, increasing intensity and consequently oscillations or/and beam break-up). Note that for 500 mW diffraction in *yz* plane is slightly better confined than in *xz* plane. It is caused by the mismatch of coupling and Rayleigh lengths.

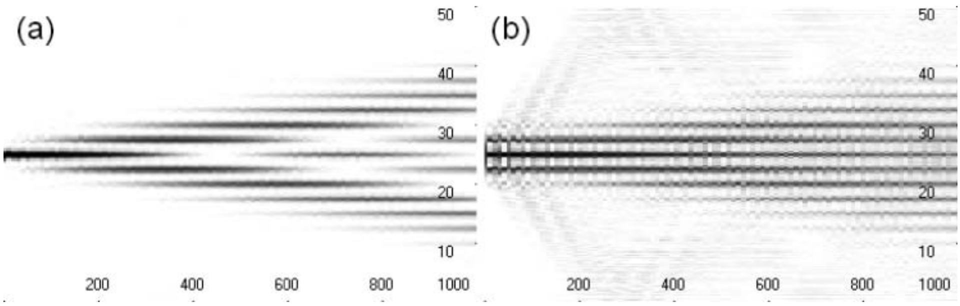

Figure 3. Comparison of light propagation (xz plane) for different beam shapes. Input power P = 1 mW, anisotropy $\Delta n = 0.06$. Beam shape: (a) elliptical $FWHM_{x/y} = 1\ \mu\text{m} \times 4 = \ \mu\text{m}$ (b) circular $FWHM_{x/y} = 4\ \mu\text{m} \times 4\ \mu\text{m}$.

For chosen anisotropy $\Delta n = 0.06$ a soliton forms for a very high input power of 500 mW. This value could be significantly decreased to about 200 mW if an elliptical beam of $FWHM_{x/y} = 1\ \mu\text{m} \times 4\ \mu\text{m}$ was used. Such elliptical beam, which width in x direction is smaller than in y direction, can be used because diffraction in the x direction does not depend on the beam width. In the linear case coupling length only depends on the pitch and anisotropy of the liquid crystal. Thus matching of Rayleigh and coupling length between orthogonal directions still stand as we change only FWHM in x direction (see Fig. 3). However, a serious drawback of such beams is that they have to be launched directly at the center of the cell. If not, huge part of the energy is strongly diffracted at the first stages of propagation. It occurs even if the beam is slightly shifted out of the center, for instance $\Delta x_s = 1\ \mu m$ (see Fig. 4). For narrow beams launched between the layers (i.e. beams of FWHM lower than half of the pitch) (Fig. 4) Floquet-Bloch modes from the second band of photonic crystal structure are excited. It is caused by the very narrow layers formed due to the low pitch of the cholesterics. In the following simulations circular beam of $FWHM_{x/y} = 4\ \mu m \times 4\ \mu m$ is used.

Simulations were also carried out for different anisotropy and different input beam power (see Fig. 5). For low anisotropy discrete diffraction is very strong and coupling length is short but it is difficult to confine the diffraction and form a soliton even for high

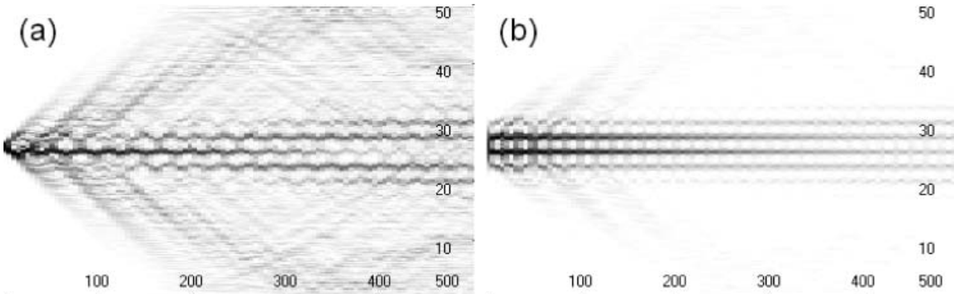

Figure 4. Comparison of light propagation (xz plane) for the beams of different shapes launched $\Delta x_s = 1\ \mu\text{m}$ below the center of the cell. Input power P = 1 mW, anisotropy $\Delta n = 0.06$. Beam shape (a) elliptical $FWHM_{x/y} = 1\ \mu\text{m} \times 4\ \mu\text{m}$ (b) circular $FWHM_{x/y} = 4\ \mu\text{m} \times 4\ \mu\text{m}$.

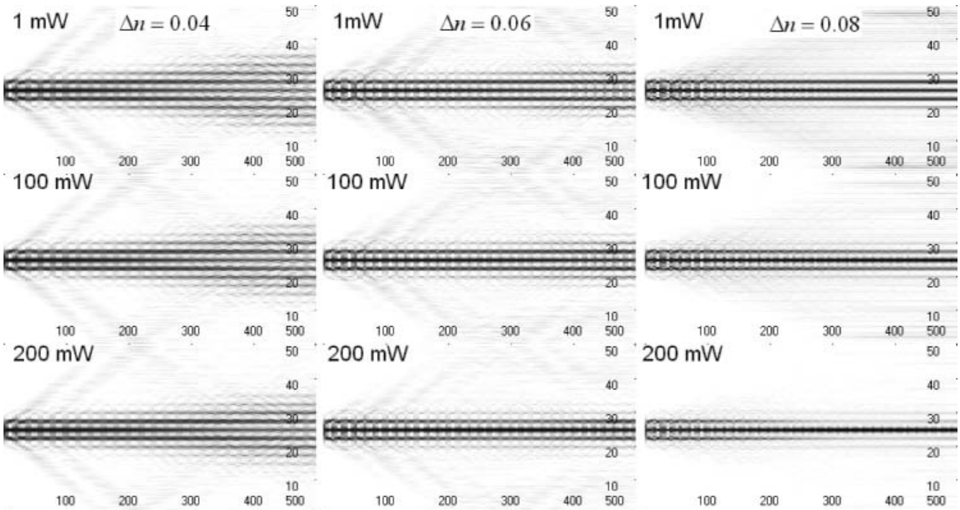

Figure 5. Light propagation (*xz* plane) for different anisotropy and input beam power.

power because the reorientational nonlinearity saturates. The higher anisotropy the longer coupling length and the narrower diffraction is. For high anisotropy it is easy to form a soliton, but discrete diffraction is very weak. Keeping in mind the above the appropriate anisotropy has to be chosen with care.

All the above simulations were performed for a beam launched straight ahead, along the *z*-axis. When a beam is launched at some low angle to the *z*-axis, the light couples from one layer to another, but with no symmetry with respect to the center of the cell (Fig. 6). Moreover, light propagates with no diffraction. Beam goes off-axis but while increasing the input power it gradually returns to the center of the cell and for a very large power (in our case between 500 mW and 1000 mW) the soliton is formed. These are extremely high powers but they can be significantly decreased by increasing the anisotropy of liquid crystals.

Conclusions

In this work we have presented numerical results of nonlinear light propagation in a cell filled with low birefriengent chiral nematic liquid crystals. Due to the low anisotropy and properly chosen pitch discrete diffraction in *xz* plane, was observed. It was also shown that light can be controlled by power and discrete diffraction can be confined. We proved that elliptical beams give better results than circular ones but need to be launched precisely at the center of the cell. It was also shown that the beam launched at some angle results in off-axis propagation with no diffraction. Such propagation can be controlled by power and for high power on-axis soliton can be formed. The presented setup can be useful for instance in building optical switches and demultiplexers. However, at first further theoretical and especially experimental results should be performed. Due to the high requirements for the light beam and small structure in comparison to the wavelength experiments have to be planned with care.

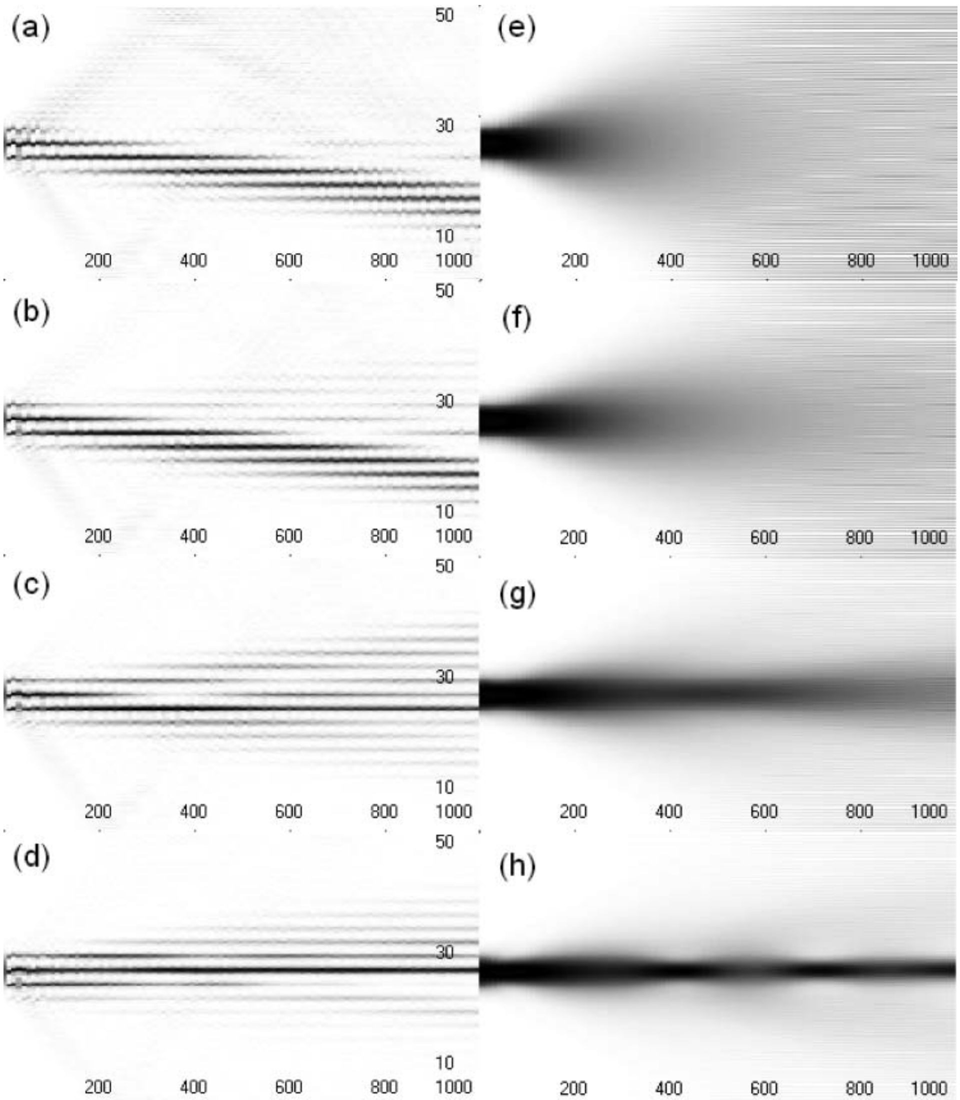

Figure 6. Light propagation in *xz* plane (left) and *yz* plane (right). Beam was launched at low angle in *xz* plane. Input power is as follows: (a)(e) 1 mW, (b)(f) 200 mW (c)(g) 500 mW (d)(h) 1000 mW.

Acknowledgments

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